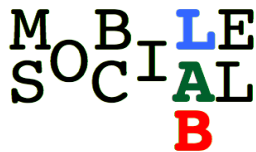


# Lecture 19: Influence (2/4)

How do influence spread?  
(i.e., how innovations get adopted?)

COMS 4995-1: Introduction to Social Networks  
Tuesday, November 27<sup>th</sup>



# Upcoming!

Yasser (How do songs become hits?)

Tikue (What's in Klout?)

Sriramkumar (Spatial Social Networks)

Jessica (What's Google new algorithm?)

Ying (How much information is fake?)

Aditya (twitter beats hedge funds?)

Chenkun (are our emotions connected?)

Yang (Twitter in the classroom)

Christopher (CrowdJobFinding)

Hooshmand (When gossip competes)

Saleh (Gender and relationships)

Guangyu (Social Networks and Health)

Xin (Facebook, Twitter and election)

Enrico (The Social Networks of Jazz)

Antony (What's Wall Street's network?)

Colin (Infection meets Computer Vision)

Xu (Fraudulent and Malicious users)

Ran (Obama's victory through big data)

# Outline

- \* Life under the influence
- \* 1978: The global view, 'Network effect'
- \* 2000: The local view, 'Neighbors effect'
- \* 2003: The algorithmic view, 'Exploiting Influence'

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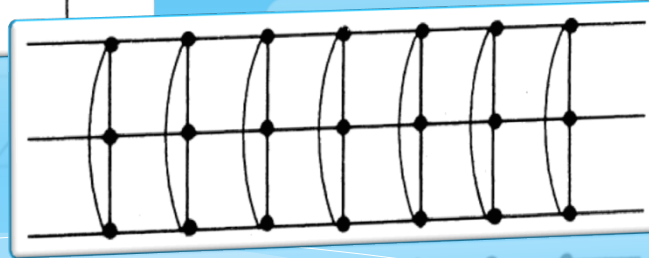
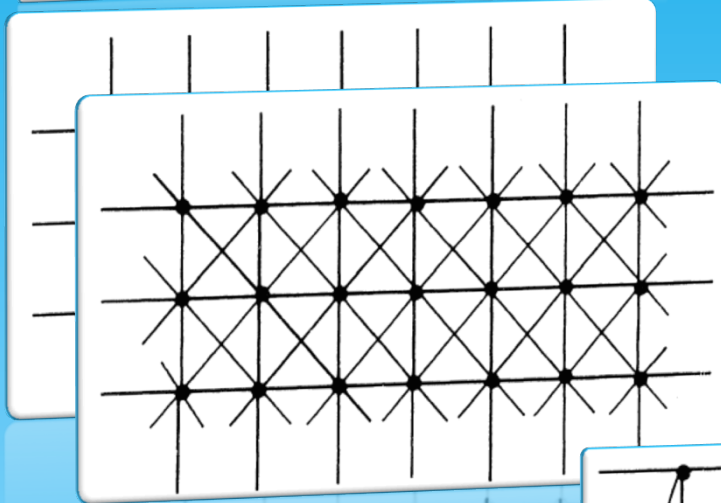
# Contagion on arbitrary graphs

- \* Previous model assumes all pairs are relevant
  - In reality, individuals know and care only about a local view of their network (i.e. fraction of neighbors)
  - E.g.: a coordination game on a graph
- \* An individual primarily decides to take an action as a monotone function of her **neighbors** adoption
  - Every node  $v$  has threshold  $t_v$
  - If a fraction  $x \geq t_v$  of her neighbors active,  $v$  activates

# Graph Conversion

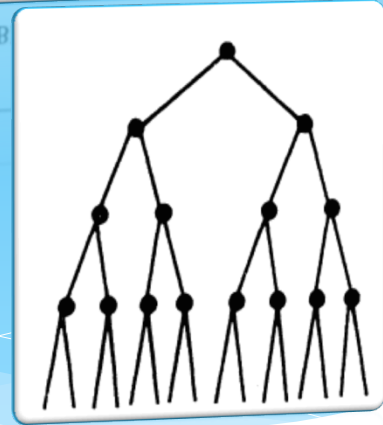
- \* Assume a finite set of seeds  $S_0$  adopting a behavior
  1.  $S_1$  nodes with fraction  $f \geq t_v$  of neighbors in  $S_0$
  2.  $S_2$  nodes with fraction  $f \geq t_v$  of neighbors in  $S_0 \cup S_1$
  3. and so on
- \* Starting from  $S_0$  the graph  $G=(V,E)$  is  $p$ -converted if
  - Assuming all nodes have  $t_v=p$ , then  $V = S_0 \cup S_1 \cup \dots$
- \* The contagion threshold  $\text{cont}(G)$  is
  - $\sup \{p \mid \text{there exists } S_0 \text{ finite s.t. } G \text{ is } p\text{-converted}\}$

# Example of $\text{cont}(G)$



	1	2	3	·	$n$	·	$n \rightarrow \infty$
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	·	$\frac{1}{2}$	·	$\frac{1}{2}$
2	$\frac{3}{8}$	$\frac{5}{12}$	$\frac{7}{16}$	·	$\frac{n(2n+1)}{(2n+1)^2-1}$	·	$\frac{1}{2}$
3	$\frac{9}{26}$	$\frac{25}{62}$	$\frac{147}{342}$	·	$\frac{n(2n+1)^2}{(2n+1)^3-1}$	·	$\frac{1}{2}$
·	·	·	·	·	·	·	·
$m$	$\frac{3^{m-1}}{3^m-1}$	$\frac{2 \cdot 5^{m-1}}{5^m-1}$	$\frac{3 \cdot 7^{m-1}}{7^m-1}$	·	$\frac{n(2n+1)^{m-1}}{(2n+1)^m-1}$	·	$\frac{1}{2}$
·	·	·	·	·	·	·	·
$M \rightarrow \infty$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{3}{7}$	·	$\frac{n}{2n+1}$	·	$\frac{1}{2}$

TABLE 1



Contagion, S. Morris, Rev. Econ. Studies (2000)

# Relation Contagion-Density

- \*  $\text{Cont}(G)$  is a graph property
  - Intuitively, contagion is difficult in denser graph
- \* A measure of density
  - A subset  $S$  is a  $p$ -dense cluster if a node in  $S$  has a fraction at least  $p$  of its neighbors in  $S$
  - $\text{clu}(G) = \inf\{ p \mid \forall |A| < \infty, V-A \text{ contains } p\text{-dense cluster} \}$
- \* Thm:  $\text{cont}(G) = \text{clu}(G)$ 
  - Dense clusters (and only them) block innovation!



# Proof of theorem

# More results on contagion

- \* What are the most favorable graph contagion?
  - In other words, how large can  $\text{cont}(G)$  be?
  - Thm:  $\text{cont}(G) \leq \frac{1}{2}$
- \* Under what conditions can it be attained?
  - At least one graph with  $\text{cont}(G)=1/2$  is known!
  - uniform interaction + low neighborhood growth
- \* What if nodes renew their decisions at each step?
  - Exactly the same: contagion threshold unchanged!

# Proof

# Proof

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